The Better Math Teaching Network:

Lessons Learned from the First Year

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Report Summary

The Better Math Teaching Network (BMTN) is a networked improvement community of researchers, teachers, and instructional leaders from New England who are using principles of improvement science to increase the number of students who are deeply and actively engaged in understanding algebra. The BMTN's quick-cycle testing involves refining and sharing student-centered instructional routines across the network. The 2016-17 school year marked the BMTN's first full year of implementation. Key findings include:

- **Strong participant uptake.** BMTN members demonstrated strong levels of buy-in and participation in the networked improvement community structure and quick-cycle testing procedures.

- **Active testing and sharing of refined routines.** BMTN teachers tested and refined a variety of student-centered instructional routines that will be shared throughout the network and beyond in 2017-18, including professional conference presentations.

- **Deepening student-centered instruction.** Teachers' choice of tasks and use of scaffolds determined the depth and degree to which students took academic ownership of the learning environment.

- **Student perceptions of classroom aligned with network aim.** A majority of BMTN students reported experiencing strongly student-centered learning environments aligned with the aim of the network, with statistically significant increasing levels from fall 2016 to spring 2017.

Who are we?

The Better Math Teaching Network (BMTN) is a community of researchers, teachers, and instructional leaders who are using student-centered instructional techniques to increase the number of students who are deeply engaged in making sense of and understanding algebra. We are organized as a networked improvement community (NIC), which means that we use principles of improvement science to develop, test, refine, and share promising instructional routines throughout the network. BMTN now has more than 60 members—most of whom are high school algebra teachers—who represent a variety of districts and schools from every state in New England. This report describes findings from the 2016-17 school year, which marked our first full year of implementation with 31 New England educators.

What problem are we trying to solve?

Far too many students are not deeply engaged in understanding and making sense of fundamental algebra content, which seriously limits their future educational and career opportunities.[1] Since Algebra I is the gatekeeper to advanced math and science coursework, this problem has broad consequences. The STEM workforce is growing fast—much faster than other job sectors—but the U.S. education system is not keeping up with this growing demand. For every approximately 200,000 new STEM jobs that are created each year, we as a nation are producing only about 20,000 qualified college graduates.[2]

While a host of factors inside and outside of schools contribute to the current situation, our network of math educators and researchers has decided to focus our improvement work on instruction. Mathematics instruction in the U.S. has been characterized as incoherent and overly procedural or rote-based[3]. This model of instruction runs counter to the kinds of problem-solving and analytical skills that are fundamental to the STEM workforce and problem-solving assessments, such as the PISA, show that many students lack these skills (see Figure 1). BMTN is trying to reverse these disturbing trends by creating algebra classrooms that are more strongly student-centered—i.e., classrooms in which students are actively making sense of mathematical content and persistently solving challenging problems.

Figure 1. U.S. Performance on the PISA in 2015
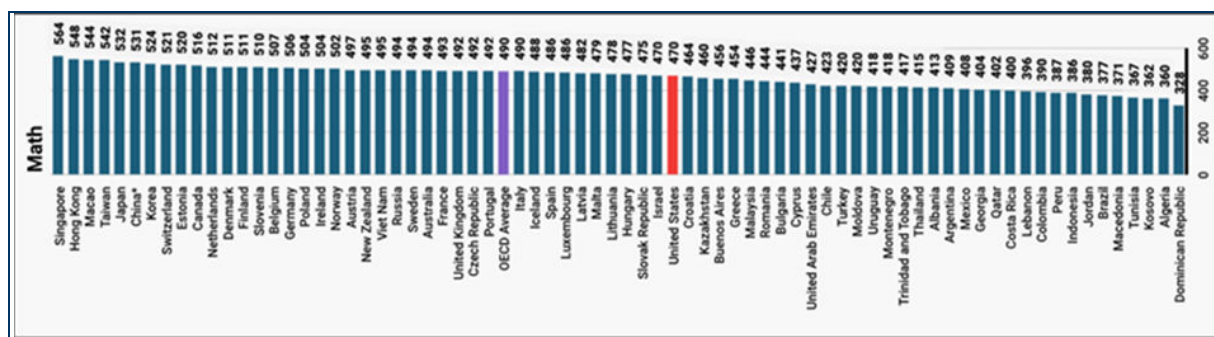
How are we trying to solve it?

BMTN is firmly committed to creating student-centered algebra classrooms in which students are actively and deeply engaged in understanding the content. But we recognize that this work is challenging. Even many well-intentioned, well-implemented reforms that seek to move beyond rote math learning fall short in producing measurable gains in student achievement.[4] We speculate that one reason why these and other methods have fallen short is because the proposed solutions have been imposed from the outside, rather than driven by the perspectives of teachers and the challenges they face on a regular basis. In fact, we view the classroom is an underutilized repository for quick-cycle testing and refinement of instructional routines, continuously informed by what teachers see day-by-day, class-by-class, in close collaboration with colleagues who are working on improving that same aspect of instruction.

More formally, we are working on this challenging problem as a networked improvement community (NIC), and we are organized according to four characteristics of well-functioning NICs.[5] These include:

- *focused* on a specific, common aim;
- *guided* by a deep understanding of the problem, the system producing it, and shared theory of how to improve it;
- *disciplined* by improvement science principles and processes; and
- *coordinated* as networks to accelerate the testing and refinement of improvement routines, their rapid diffusion and integration to varied instructional contexts.[6]

Focused on a common aim

One of the hallmarks of a NIC is that it is anchored in a common aim. BMTN is focused on creating student-centered learning classrooms in which more students are deeply and actively engaged in making sense of and understanding algebra. Yet, this aim is too broadly defined to facilitate the testing and refinement of specific student-centered instructional routines. Thus, as a network, and informed by the best available research[7] including some of our own work[8], we have narrowed our focus to three specific forms of deep engagement. These are making deep mathematical connections, justifying and critiquing mathematical thinking, and solving challenging problems, each of which is elaborated below:

- **Connect.** Making connections among mathematical algorithms, concepts, and application to real-world contexts, where appropriate.
- **Justify.** Communicating and justifying mathematical thinking as well as critiquing the reasoning of others.
- **Solve.** Making sense of and solve challenging math problems that extend beyond rote application of algorithm.

Reflecting a diverse New England instructional setting, which includes teachers and algebra students from urban, rural, and suburban schools, our formal aim statement (below) incorporates these three forms of deep engagement. As illustrated in the text box to the right, BMTN's common goal is to increase the number of New England students who connect, justify, and solve with depth by 2,019 students by the end of 2018-19 school year. This number reflects our current assumptions about the number of algebra students assigned to BMTN teachers and may be adjusted upward as more teachers join the network. As we test and refine student-centered instructional routines, we track progress against our aim statement.

> **BMTN Aim Statement**
> By 2019, the number of students who connect, justify, and solve with depth will increase by 2,019.

Guided by a working theory of improvement

This aim statement is reflected in our network's working theory of improvement, or driver diagram. A driver diagram is a tool that helps organize ideas and theories of improvement[9]. It is designed to be a living document that is updated throughout the life of an improvement project, informed from what the network is learning through ongoing testing. BMTN's current driver diagram, illustrated in Figure 2, has gone through several iterations.

Figure 2. BMTN Driver Diagram
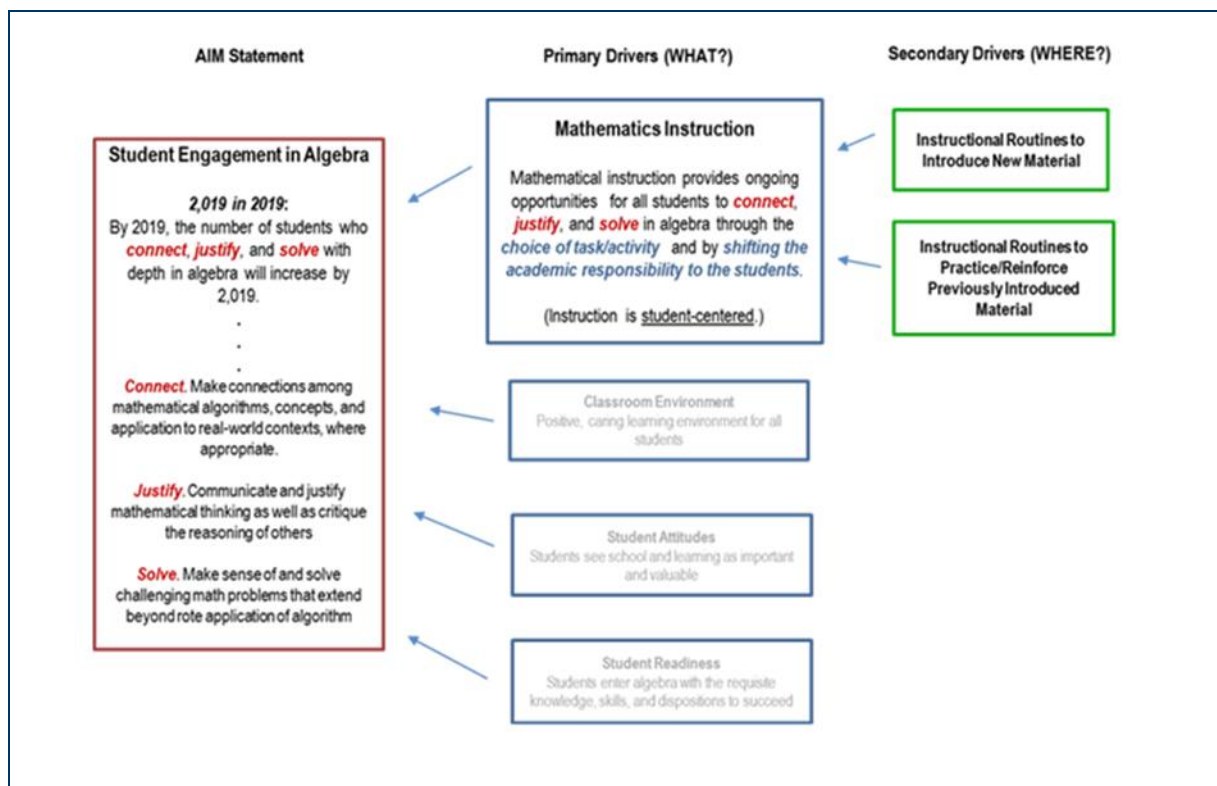
BMTN assumes that deep student engagement in learning algebra is influenced by several primary drivers, but given our focus on teachers and teaching, most of our work emphasis is on the driver of mathematics instruction. That is, teachers can create more opportunities to solve, justify and connect with depth by how they orchestrate the classroom. During the 2016-17 school year, we identified two common threads in the instructional environment that influenced the degree to which it fostered, deep student-centered learning: the tasks that teachers use with students, and the extent to which teachers shift the academic ownership of learning the content to the students.

Our driver diagram also has two secondary drivers, which further specify where changes might be tested and refined. These are related to the nature of the mathematics content: whether students are being exposed to the content for the first time or whether they are practicing or reinforcing content to which they have already been exposed. Taken together, the driver diagram illustrates why specific types of improvement routines might lead to specific kinds of outcomes in support of BMTN reaching its aim of increasing the number of students who connect, justify, and solve with depth.

Disciplined inquiry using improvement science

The aim statement and driver diagram organize the day-to-day work of the network, which involves the rapid testing and refinement of specific change ideas, which in our case, are

instructional routines. Such quick-cycle testing is based on principles and processes from improvement science—a method that has been used increasingly over the past century in manufacturing, but is relatively new to education[10]. Figure 3 illustrates the framework BMTN is using to drive its rapid-cycle testing of student-centered instructional strategies. We ask three essential questions for any improvement effort: (1) What am I trying to accomplish? (2) What changes will I make and why? and (3) How will I know that the change actually led to an improvement? These questions are reflected in the formal testing of change ideas, which follow Plan-Do-Study-Act (PDSA) cycles of improvement. Each PDSA cycle is intended to "turn ideas into action and connection action to learning."[11] That is, BMTN members create a specific **plan** for a change idea that is attached to the Driver Diagram, **do** or carry out the plan (e.g., implement the student-centered instructional routine and collect data on what happened), **study** the data to summarize what was learned, and **act** according to the data (e.g., abandon, adapt, or adopt the strategy as implemented).

Figure 3. BMTN Framework for Ongoing Testing of Instructional Routines

> **What am I trying to accomplish?**
> **What changes can I make and why?**
> **How will I know the change is an improvement?**
>
> ACT | PLAN
> STUDY | DO

LeMahieu et al., 2017; Langley et al., 2009
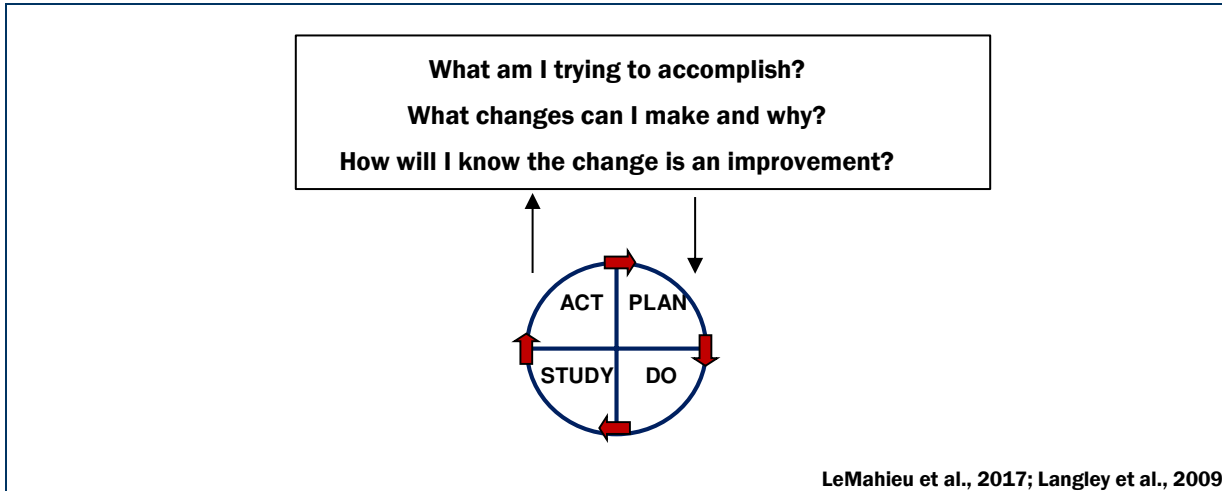
During the 2016-17 school year, BMTN teachers used the PDSA process to test and refine a number of different instructional routines to support the network in reaching our aim. Thus, the routines were aligned with our driver diagram, both the primary and secondary drivers. More specifically—and to support the testing of fine-grained strategies—each routine was focused on one of the three forms of deep student engagement (connect, justify, solve) in our primary driver and or both of the secondary drivers (introduction to new material and reinforcement of previously introduced material). Table 1 provides a brief description of some of the routines that were refined.

Table 1. Instructional Routines Refined Through PDSA Testing, 2016-17 BMTN Teachers*

Routine	Primary Driver			Secondary Driver	
	Connect	Justify	Solve	Intro	Reinf.
Student discourse protocol to elicit mathematical connections	✓			✓	
Open-ended problems to connect new to prior knowledge	✓			✓	
Exit tickets that assess connections to be addressed next day	✓				✓
Exit tickets to support developing connections	✓			✓	✓
Exit tickets to develop connections to prior knowledge	✓			✓	
Written examples to help students improve problem solving			✓		✓
Structured routine to help students solve challenging problems			✓	✓	✓
Written protocol to promote student reflection on homework			✓		✓
Protocol to help students self-monitor during problem solving			✓		✓
Problem-solving routine to support written justification		✓			
Formative assessment routine to promote justification		✓			✓
Claim-evidence-reasoning protocol to deepen justification		✓			✓
Teacher questions and student prompts to promote justification		✓		✓	✓
Open-ended tasks with discussion routine to support justification		✓		✓	
Student errors and stuck points to promote justification		✓		✓	✓
Formative assessment strategy to deepen justification		✓		✓	✓
Adapting a student discussion protocol to deepen justification		✓			✓

* Data taken from BMTN teacher change idea summaries, 2016-17 school year.

Are we making progress?

NICs develop in three phases.[12] They begin with a **chartering** phase, where a small team of improvement scientists, content experts, and practitioners identify the problem to be solved, examine system-based causes of the problem, and establish visual tools, including a driver diagram, to orient the collective improvement work of the network. The BMTN completed its chartering phase during the 2015-16 school year. After the chartering phase, NICs enter into a period of **network learning** in which network members use PDSA cycles to test and refine different types of routines, share replicable strategies with each other, and use common measures to assess whether and where improvements are happening. The BMTN moved to this phase during the 2016-17 school year. The final phase of NIC development is **spreading** the knowledge gained through the network with other individuals and organizations. We will enter this final phase during the 2017-18 school year.

In the following sections, we present three sets of findings from our network learning phase. The first two sets relate to the ongoing work of the network: the testing and refinement of student-centered instructional routines, and the network structures that are facilitating this work. The third set of findings describes summative progress we are making towards reaching our network aim.

Learnings related to student-centered instruction

Through multiple rounds of testing and refining student-centered instructional routines, our teachers learned that, to increase the numbers of students who deeply engage with algebraic content, they need to:

1. Use tasks that provide opportunities for students to deeply engage in making connections, providing justifications, and solving non-rote problems;
2. Provide scaffolds to support students in making connections, providing justifications, and solving non-rote problems at a deep level; and
3. Remove the scaffolds over time to promote student ownership of the material.

The specific types of tasks and scaffolds needed to support student sense-making and understanding varied by area of deep engagement. That is, some teachers focused on instructional routines to promote connections, other on routines to promote justification and, still others, on routines to promote problem solving.

Teachers who tested routines to promote student agency in making connections learned that the choice of tasks was integral to (a) supporting students in making deep connections and (b) determining whether or not students had made those connections. In particular, they learned that the tasks need to elicit connections between mathematical algorithms and concepts or connections between two different mathematical concepts. To choose or develop such tasks, our teachers learned that they themselves need to be able to clearly articulate the deep connection they are hoping to elicit from students. Finally, our teachers observed that they need to provide scaffolds to support students working with these rich tasks. Sample scaffolds include: guiding questions to support students in discovering a new connection, sentence starters to support students in articulating a connection they made, and activities to encourage analysis of sample explanations to support students in better articulating the connection they are making. Box 1 highlights this learning.

Box 1. Lessons Learned from Katie's Exit Ticket Routine (Connect)

Problem and Initial Change Idea: Katie typically uses inquiry-based activities s to introduce new material and support the development of student understanding. These tasks tend to require the full class period (or several class periods) to complete, and students typically worked on them up until the end of the class period, with no time for reflection. To better support her students' developing understanding of the connections the activities are meant to elicit, Katie wanted to provide an opportunity for them to pause at

the end of the class period and reflect on the connections between mathematical algorithms and the concepts that were elicited during the activity.

Testing: Katie initially tested a routine in which students were given an exit ticket that asked students to (1) describe the mathematical concept they learned that day, (2) list questions they had about the concept, (3) identify a misconception they or a classmate had, and (4) describe what they learned from resolving that misconception. After several rounds of testing, Katie learned that, in order to support reflection on connections and assess the extent to which students made those connections, she needed to modify the questions on the exit ticket. In particular, she needed to explicitly ask students to identify the connections, not simply state what they learned. She also found that asking the same question over the course of several days supported her students in solidifying their understanding of the connections. In order to write the questions, though, Katie needed to articulate the connection she was hoping students to make for herself.

Katie tested the new questions and learned that, although students could articulate the connections verbally during the lesson, they had difficulty doing so in writing on the exit tickets. She further refined the routine to include an activity in which students analyze sample explanations to come to a common understanding of what constitutes a strong explanation. Over time, she removed this scaffold.

Evidence of Promise: By the end of testing, more than 86 percent of students were showing improvement in making deep connections and being able to articulate those connections.

Context: Katie teaches in an urban environment with students who struggle with substance abuse. More than half of her students are in special education.

Like those testing routines focused on connections, teachers who tested routines focused on justification and critique learned that they needed to provide tasks that were rich enough to elicit strong mathematical justifications and critiques. Tasks that require students to support their reasoning by appealing to mathematical concepts, as opposed to "rules", seemed to work best. In addition, these teachers learned that they need to scaffold the work with the tasks over time so that students first become comfortable with sharing their thinking, then develop skill in providing strong justifications for their thinking and, finally, develop skill in providing strong critiques for others' thinking. Box 2 highlights this learning.

Box 2. Lessons Learned from Jacob's Problem-Solving Routine (Justify)

Problem and Initial Change Idea: When Jacob asked his students to justify their answers and evaluate the answers of their peers, his students tended to shut down and become disengaged. He decided to use open-ended problems paired with a structured discussion protocol to promote communication, justification, and critique.

Testing: Jacob tested a routine in which students are first given an open-ended problem and asked to work on the problem individually. After a few minutes, they share their thinking with a partner, hear their partner's thinking and, potentially, revise their thinking. One student is randomly chosen to share his/her thinking with the whole class and the other students discuss the validity of the approach with their partner. The cycle of student presentation and partner discussion continues until students are satisfied with the solution or more instruction is needed.

Because he worried that students would shut down if given algebra problems during this routine, Jacob started by using Bongard problems, which require reasoning but are not tied to specific algebraic concepts. Once Jacob was satisfied with student progress in working with the Bongard problems, he began to introduce algebra problems into the routine. Because students had gained confidence in their ability to justify their thinking and critique others with non-algebra problems, they were able to make the transition to algebra problems. Eventually, he used the problems to introduce new material, as opposed to simply practice previously learned material.

As Jacob made the transition to algebra problems, he noticed that students were able to provide strong justifications for their own thinking but the quality of the critiques they gave each other were not strong. He therefore provided a series of guiding questions to support students in providing quality critiques and their critiques began to improve.

Evidence of Promise: At the end of testing, more than 90 percent of Jacob's students were providing strong justifications for algebra problems and close to 80 percent were writing quality critiques. This shows great improvement as he started with only 20 percent even attempting to engage in justification activities.

Context: Jacob works in a rural district. Over half of his students qualify for free or reduced-priced lunch. He tested his change idea in a low-achieving algebra class. Half of the students in that class have Individualized Education Plans or 504 plans.

Finally, like the teachers focused on connecting and justifying, teachers who tested routines to promote perseverance and success in solving non-routine algebra problems learned that the choice of problems was integral to developing these skills in students. The problems needed to be rich enough to promote deep work with the content and require student perseverance to solve. To support students in working with these problems, teachers learned that they needed to provide protocols or routines to encourage student reflection on their problem-solving approaches along the way. These include requiring students to make sense of the problem and asking them to share their thinking as they work towards a solution. Box 3 illustrates this learning.

Box 3. Lessons Learned from Jenny's Formative Assessment Routine (Solve)

Problem and Initial Change Idea: Jenny realized that too much of her teaching focused on teacher demonstration followed by students repeating her actions. She wanted her students to be able to work on and solve non-rote math problems without her first demonstrating how to solve them. When she would give such problems, her students would shut down. She decided to implement a new formative assessment problem-solving routine to give her students more opportunities for work with such problems and reflect on the problem-solving process in hopes that they would develop the confidence and skill needed to persevere in working on the problem.

Testing: Jenny initially tested a routine in which she gave students an unfamiliar problem and used a structured protocol whereby students examined the problem, chose a strategy to solve the problem, and worked with a partner to solve it. After a period of individual and partner work, students were asked to explain what their solution means in the context of the problem or, if they didn't solve it, explain what prevented them from reaching an answer. After several rounds of testing, Jenny learned that she needed to provide more structure to the routine to support student engagement in problem solving and reflection. In particular, she needed to provide worksheets with the problem for individual work and, then again, for work

with a partner. In that way, students could reflect on their own work and the work of the group. She also learned to provide a separate reflection sheet for students to make sense of the solution or provide an explanation of the challenged faced in solving the problem. Finally she added whole-class discussion of the process to support students in developing confidence and problem-solving strategies.

Evidence of Promise: Prior to testing, none of Jenny's students were able to identify a strategy to solve non-routine problems. By the end of testing, close to 80 percent of students could identify a strategy and attempt to use that strategy. This suggests that they were beginning to develop confidence and skill in beginning non-routine problems.

Context: Jenny teaches in a mostly white, rural district. More than half of the students qualify for free and reduced-price lunch.

Learnings related to network structures

In addition to learning about instructional routines, we learned a great deal about how to structure BMTN's work. With 23 teachers and 8 instructional leaders, it can be challenging to coordinate and support the work, summarize the learnings, and provide opportunities for sharing both within and beyond our network. In our first year, we found success with the following:

- Structuring the work as a series of small-group PDSA meetings focused on a common element of deep algebra engagement combined with periodic whole-group meetings to address challenges and share learnings;
- Providing a "Change Idea Summary" template for teachers to summarize and share what they learn across multiple PDSA tests of their change idea;
- Organizing an end-of-year, full-network meeting with teachers, hub members, and instructional leaders designed to share, summarize, and celebrate the learning done throughout the year; and
- Supporting teachers in preparing conference proposals and presentations.

Small Group PDSA Meetings

To support teachers in testing and refining their changes ideas and provide opportunities for whole-network learning, teachers met as smaller PDSA testing groups using video conferencing every four to six weeks. In addition to these smaller group meetings, the whole network met in-person at four time points during the year, drawing from work done in smaller groups between the larger meetings. Each small PDSA group included three or four teachers focused on the same DEA and similar change ideas. One hub member worked with each PDSA group to provide coaching on improvement science and expertise in math education. Prior to each group meeting, teachers were expected to have tested their change idea, completed the PDSA form, and shared the form with the other members on a common shared drive. During the meeting, the group discussed each teacher's testing and next steps. During the whole network meetings, teachers shared and synthesized learning across PDSA groups and addressed challenges faced in PDSA testing. For

example, when teachers struggled to measure deep engagement with algebra concepts, we spent time in the whole network meeting to develop a shared understanding of deep engagement and discuss ways of measuring it. Teachers were able to apply their learnings to future PDSA testing. Overall, this structure seemed to work well for supporting individual and whole-group learning.

Change Idea Summaries

As teachers implemented multiple PDSA tests, we found that they needed a way to synthesize the learning across tests both for themselves and to share with the broader network. To support this work, we developed a Change Idea Summary template where teachers provided an overview of the problem, described the change idea they tested, offered a detailed description of the routine they would recommend after refinement through testing, included data to show evidence of promise, and gave additional advice for implementing the change idea. Teachers were encouraged to attach resources, such as sample tasks, and measures that would be useful for implementing and testing the change idea to the Change Idea Summary. The Change Idea Summaries were then combined into a book to share with the full network. Not only did the Change Idea Summaries support teachers in synthesizing and sharing their learnings but they also provided a means by which we could celebrate the work of BMTN and, potentially, spread the work beyond our network.

End-of-Year Sharing

Our last in-person, whole-network meeting was used to share the Change Idea Summaries and celebrate the work. We invited the teachers as well as the instructional leaders to the meeting and divided them into three groups so that there was teacher and instructional leader representation in each group. Teachers used a Power Point template that we provided to present to and answer questions from their group about their Change Idea Summaries. At key points during the day, we conducted whole-group conversations to synthesize learnings across the three groups. The meeting ended with cake, champagne, group reflections on the year, and a group picture, putting a celebratory exclamation point on our collective work. The instructional leaders provided positive feedback on the quality of the teacher presentations and eagerness to help move the work forward in the 2017-18 school year.

Teacher Professional Presentations

In addition to providing opportunities for internal sharing within the network, we provided technical assistance to teachers to share their learnings at conferences during the school year and this support was well-received among the teachers. Over the course of the year, several BMTN teachers presented at state and regional math teacher meetings. We provided feedback to these teachers on their proposals and presentations. We also incorporated teachers' change ideas and classroom artifacts (e.g., written excerpts of students' work and audio recordings of oral work) into two BMTN presentations at national conferences. We plan to do more of this during the year. Not only does the approach provide support to teachers who many have not otherwise decided to present, it provides opportunities for others outside of BMTN to learn about our work and helps enhance the teaching profession.
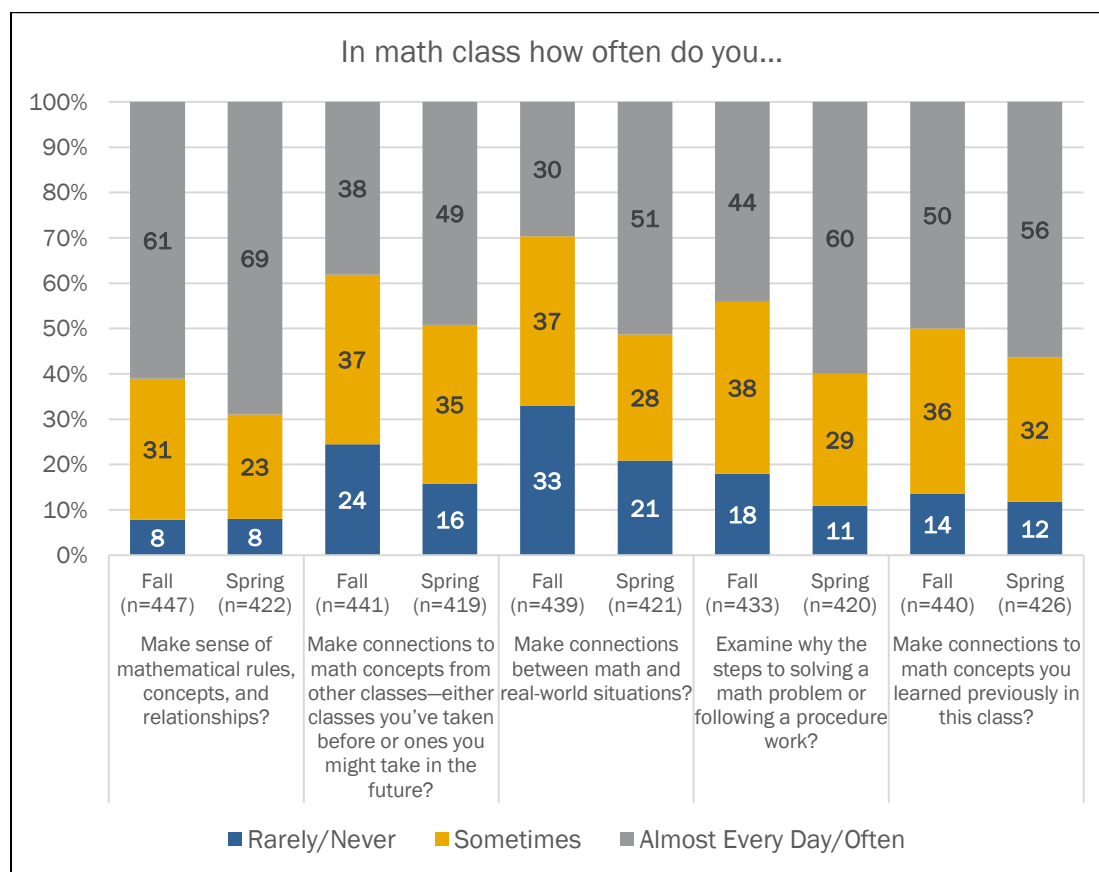
Progress towards our network aim

In addition to the lessons we are learning through the iterative, rapid-cycle testing occurring throughout BMTN, we are also tracking progress against our aim. By 2019, we want the number of New England algebra students who connect, justify, and solve with depth in algebra to have increased by 2,019.

One way we are measuring this progress is through a student survey. We are interested in hearing from students throughout the year—in fall, winter, and spring—about the extent to which they are connecting, justifying, and solving with depth in algebra. The following figures show progress on this front. Overall, the trends are positive.

Figure 4 illustrates that students report making deep mathematical connections with more frequency throughout the 2016-17 school year. The differences between student reports in the fall and spring were statistically significant ($p < 0.05$) for each question in the figure. For example, the percentage of students who reported making connections between math and real-world situations often or on a daily basis increased from 30 percent in the fall to 51 percent in the spring. Similarly, there was a 16 percentage point increase in the extent to which students were examining why steps to a math problem work—from 44 percent in the fall to 60 percent in the spring. In addition to the positive trends at the often or almost daily levels, the data show that the percentage of students who never or rarely connect in these ways is quite low—between 8 percent and 21 percent in the spring for all five questions. The 21 percent applies to making connections between math and the real world. These connections do not lend themselves to every lesson so we would expect that a greater percentage of students would indicate that they never or rarely make these types of connections. The fall-to-spring increase of 30 percent to 51 percent for this category still suggests growth in this area.
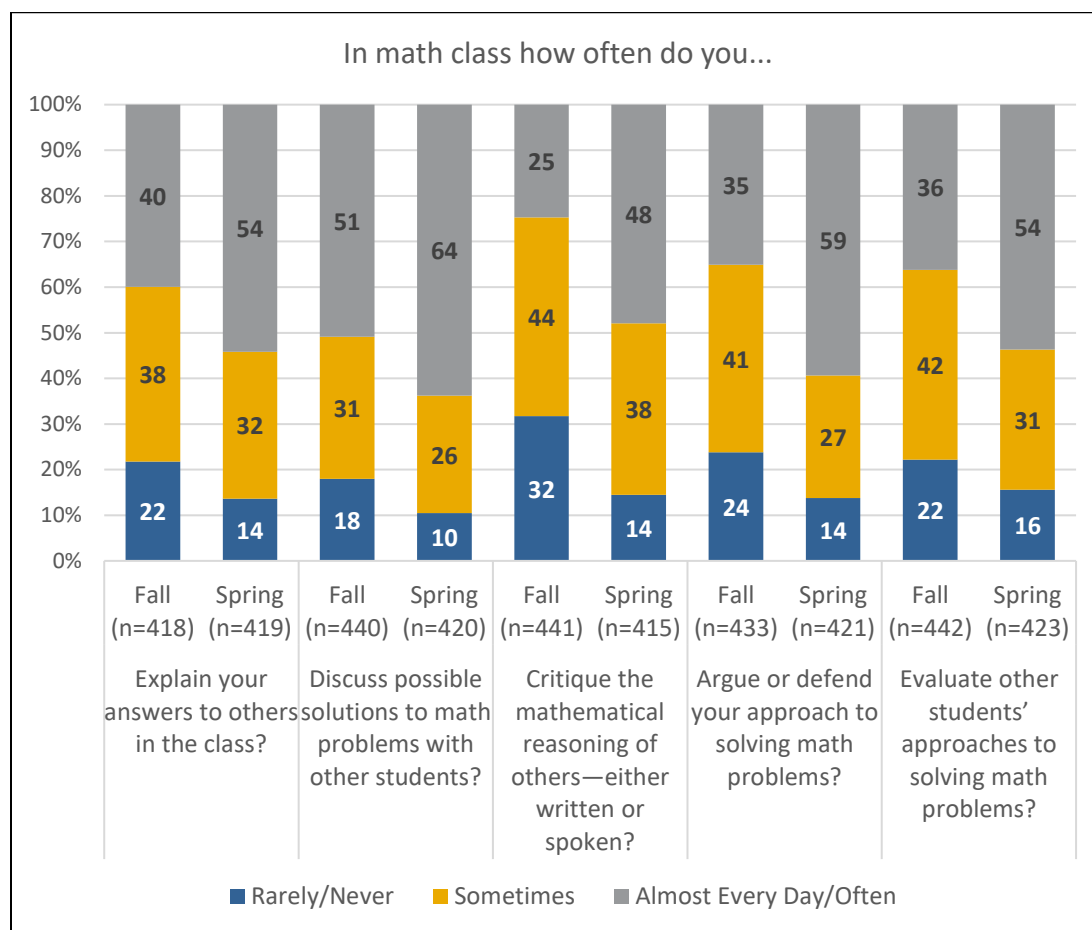
Figure 4. Student Reports of Making Mathematical Connections, 2016-17

In math class how often do you...

	Fall (n=447)	Spring (n=422)	Fall (n=441)	Spring (n=419)	Fall (n=439)	Spring (n=421)	Fall (n=433)	Spring (n=420)	Fall (n=440)	Spring (n=426)
Almost Every Day/Often	61	69	38	49	30	51	44	60	50	56
Sometimes	31	23	37	35	37	28	38	29	36	32
Rarely/Never	8	8	24	16	33	21	18	11	14	12

Questions: Make sense of mathematical rules, concepts, and relationships? | Make connections to math concepts from other classes—either classes you've taken before or ones you might take in the future? | Make connections between math and real-world situations? | Examine why the steps to solving a math problem or following a procedure work? | Make connections to math concepts you learned previously in this class?

■ Rarely/Never ■ Sometimes ■ Almost Every Day/Often

Note: The number of students who took the survey varied with the number of teachers who administered the survey to their students at each data point (Fall = 20 teachers, Spring = 18 teachers). Additionally, the number of students who responded to each survey question at each data point varies because some students did not respond to every survey question.

Students also reported a steady increase in the degree to which they made mathematical justifications with depth during the 2016-17 school year, illustrated in Figure 5. Three of these five increases from fall to spring were statistically significant (p<0.05) and quite large. For example, the percentage of students critiquing the mathematical reasoning of others often or almost daily more than doubled—from 25 percent in the fall to 51 percent in the spring. Similarly, 54 percent of students reported evaluating other students' approaches to solving problems often or almost daily, which was almost 20 percentage points higher than the 36 percent reported in the fall. The results also show a marked increase in the percentage of students reporting that they frequently argue or defend their approach to solving math problems. Thirty-five percent reported doing so often or almost every day in the fall compared to 60 percent who said they did so in the spring. As with the making connections data, the percentage of students who never or rarely justify in these ways is low—between 11 percent and 15 percent in the spring for all five questions.

Figure 5. Student Reports of Justifying Mathematical Thinking, 2016-17

In math class how often do you...

	Fall (n=418)	Spring (n=419)	Fall (n=440)	Spring (n=420)	Fall (n=441)	Spring (n=415)	Fall (n=433)	Spring (n=421)	Fall (n=442)	Spring (n=423)
Almost Every Day/Often	40	54	51	64	25	48	35	59	36	54
Sometimes	38	32	31	26	44	38	41	27	42	31
Rarely/Never	22	14	18	10	32	14	24	14	22	16

Explain your answers to others in the class? | Discuss possible solutions to math problems with other students? | Critique the mathematical reasoning of others—either written or spoken? | Argue or defend your approach to solving math problems? | Evaluate other students' approaches to solving math problems?

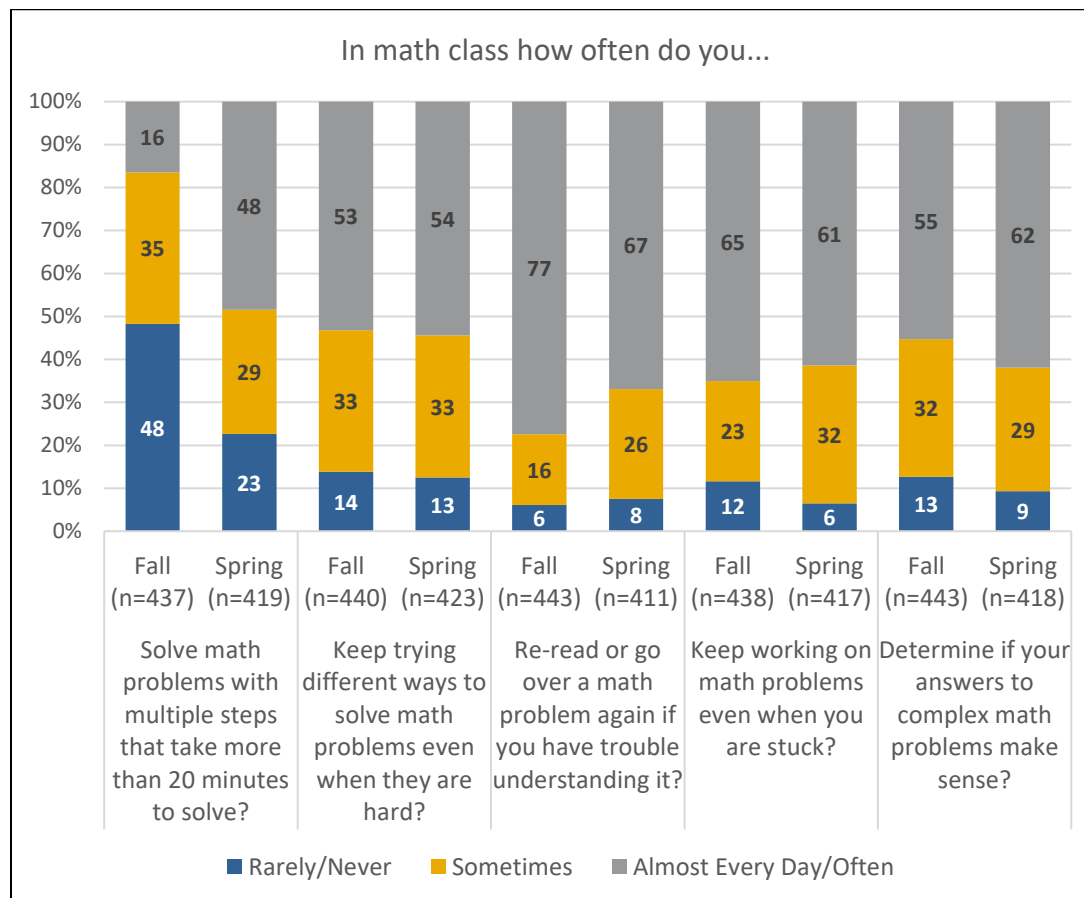■ Rarely/Never ■ Sometimes ■ Almost Every Day/Often

Note: The number of students who took the survey varied with the number of teachers who administered the survey to their students at each data point (Fall = 20 teachers, Spring = 18 teachers). Additionally, the number of students who responded to each survey question at each data point varies because some students did not respond to every survey question.

Finally, as shown in Figure 6, the percentage of students who reported solving with depth (the third DEA) increased in some areas over the 2016-17 school year. The differences between student reports in the fall and spring were statistically significant ($p<0.05$) for each question in the figure. For example, the percentage of students who reported regularly solving math problems with multiple steps that take more than 20 minutes almost tripled—from 17 percent in the fall to 51 percent in the spring. There was no change in the percentage of students who reported trying different ways to solve problems often or almost every day, and a modest increase in the percentage of students who said they determine if their answers to complex problems make sense regularly (55 percent in the fall vs. 62 percent in the spring). Finally there were moderate decreases in the percent of students who reported that they keep working on problems when they are stuck (65 percent in the fall vs. 61 percent in the spring) and re-read or go over the problem again when they have trouble understanding it (78 percent in the fall vs. 67 percent in the spring). Though these decreases are modest, and a majority of students still reported solving in these ways in the spring, they suggest that supporting students with persisting in solving

challenging problems is an area for continued work. These data also correspond to students reporting that they are solving longer, multi-step problems in the spring, so it may be worth further exploring issues around the frequency and length of these longer problems.

Figure 6. Student Reports of Solving Challenging Problems, 2016-17

In math class how often do you…

	Fall (n=437)	Spring (n=419)	Fall (n=440)	Spring (n=423)	Fall (n=443)	Spring (n=411)	Fall (n=438)	Spring (n=417)	Fall (n=443)	Spring (n=418)
Almost Every Day/Often	16	48	53	54	77	67	65	61	55	62
Sometimes	35	29	33	33	16	26	23	32	32	29
Rarely/Never	48	23	14	13	6	8	12	6	13	9

Categories: Solve math problems with multiple steps that take more than 20 minutes to solve? | Keep trying different ways to solve math problems even when they are hard? | Re-read or go over a math problem again if you have trouble understanding it? | Keep working on math problems even when you are stuck? | Determine if your answers to complex math problems make sense?

Legend: ■ Rarely/Never ■ Sometimes ■ Almost Every Day/Often

Note: The number of students who took the survey varied with the number of teachers who administered the survey to their students at each data point (Fall = 20 teachers, Spring = 18 teachers). Additionally, the number of students who responded to each survey question at each data point varies because some students did not respond to every survey question.

Where are we headed next?

The fourth stage of NIC development involves spreading or diffusing knowledge that is being generated by the network. For BMTN, this knowledge includes routines that have been tested and refined with smaller groups of teachers, and are now ready to be shared with other teachers within the network and others outside the network who are addressing a common problem. In a NIC, the principle of "adaptive integration" applies to how knowledge spreads.[13] This principle assumes that there will be variation in how the routine or strategy is implemented in each context. Thus, the goal is to understand whether and how such variation occurs. This approach is different from frameworks that emphasize implementing a particular routine or strategy with fidelity. Here,

variation is expected and the knowledge that is gained from implementing the strategy in varied contexts adds to the collective learning of the NIC and contributes to achieving the aim.

During the 2016-17 school year, BMTN teachers spread instructional routines informally, within their smaller PDSA testing groups and through the whole-network meetings. Such spread occurred when teachers heard about a new routine in one of these venues and then followed up with the teacher who developed the routine to learn more about it and determine whether and how to integrate it into their respective classrooms.

The sharing and spreading of refined routines has begun to occur more formally and will continue during the 2017-18 year. In May, we shared the set of routines that teachers refined during the 2016-17 school year with BMTN teachers and instructional leaders. The instructional leaders plan to share these strategies with the teachers and other instructional leaders in their jurisdictions, providing additional avenues for the refined strategies to be shared. In July, new and returning teachers reviewed the refined routines together, and several have selected one of these routines as the focus of their first round of PDSA testing in the 2017-18 school year. Finally, in fall 2017, the refined routines will be made available on the BMTN website and linked to on its social media platforms, providing other channels for this knowledge to be shared outside the network.

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